Two-Dimensional Jet Interaction Studies at Large Values of Reynolds and Mach Numbers

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The situation chosen for study is a gaseous jet that is injected through a transverse slot nozzle in a wall and into a supersonic external flow which is uniform outside of a turbulent boundary layer. Experiments were conducted with normal, sonic jets and forward-facing steps at external flow Mach numbers of 2.5 to 13, and Reynolds numbers based on running length of 7.5×10^6 to 5.5×10^8 . The amplification factor (the upstream interaction force plus the jet thrust normalized by the vacuum thrust of a sonic jet) is relatively insensitive to variations in external flow Mach number and Reynolds number. The effect of pressure ratio on amplification factor is very small when the external flow properties and jet mass flow rate are held constant. Plateau pressures associated with separation upstream of the jet or step, and wall static pressure distributions near the separation line are in good agreement with existing correlations.

Nomenclature

\boldsymbol{c}	= nozzle discharge coefficient
d	= slot nozzle width
F_{i}	= interaction force, upstream of jet nozzle
h	= forward-facing step or spoiler height
K	= amplification factor = $(F_i + T)/T_{sv}$
L	= distance along the x-axis from the origin of the boundary layer to the nozzle centerline
L_s	= distance along the x-axis from the origin of the
	boundary layer to the separation line
M	= Mach number
\boldsymbol{P}	= pressure
Re	= Reynolds number
T	=thrust
T_{sv}	=vacuum thrust of a sonic jet
x	= coordinate parallel to solid surface, aligned with the external flow
y	= coordinate transverse to flow direction
Δx_s	=x measured from separation point
δ	= boundary layer thickness
Subscript	S
0	= stagnation conditions
1	= region upstream of separation outside of the boundary layer
2	= region downstream of separation shock, at edge of shear layer
j	=refers to the jet

Introduction

THE flowfield that results from the interaction between a gaseous jet and a supersonic external flow, sometimes called the jet interaction flowfield, has been the subject of numerous investigations. The literature related to this topic has been reviewed by Spaid and Cassel. This type of interaction is associated with thrust vector control of rocket motors by gas injection, supersonic combustion of a gaseous fuel, and jet interaction control of high-speed flight vehicles.

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Because of its complexity, knowledge of these interactions has been derived primarily from the results of experiments. The purposes of the present experiments were to extend the body of two-dimensional jet interaction data to higher Mach numbers and Reynolds numbers, to investigate the independent influence of pressure ratio on jet interaction performance, and to obtain static pressure data for turbulent boundary-layer separation at high Reynolds numbers.

Before discussing the details of the experiments, the qualitative features of the jet interaction flowfield will be reviewed briefly. Figure 1 is a schematic diagram of a typical flowfield with the associated static pressure distribution. In this example, the jet is sonic, underexpanded, and normal to the wall. The boundary layer is turbulent upstream of the interaction region, and the effective obstruction to the external flow formed by the jet is larger than the undisturbed boundary-layer thickness. The boundary layer is separated upstream of the jet location, and a shock wave originates near the separation line. The static pressure rises in the vicinity of separation, reaches a plateau (data from some experiments show a first peak), and rises again in the immediate vicinity of the jet. Another shock wave originates in the region where the separated boundary layer and the shear layer at the upstream boundary of the jet meet and form the mixing layer between the jet and the external flow.

Another separated region exists downstream of the jet and has some of the characteristics of the flow over a rearward-facing step. When the external flow is supersonic, the static pressure immediately downstream of the jet is less than the static pressure of the undisturbed flow, P_1 . In hypersonic

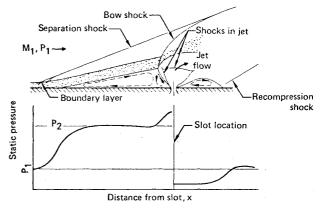


Fig. 1 Sketches of flowfield and pressure distribution.

flow, downstream pressure distributions in which P/P_1 is always greater than unity are often observed.

Description of Experiments

Three sets of experiments were conducted. In each case, a gas was injected through a transverse slot in the surface of a flat plate or test-section wall model, normal to the external flow. Some tests were also conducted with solid spoilers installed at the nozzle location. End plates were mounted at both ends of the jet nozzle or spoiler.

The use of end plates to achieve two-dimensionality in separated flow studies has been discussed extensively in the literature; however, it is still not possible to determine accurately the degree to which a particular end-plate design will be successful in eliminating three-dimensional effects. In a brief study of end-plate effects in connection with separation of a turbulent boundary layer on a wind tunnel wall by a forward-facing step, it was noted that the wall static pressure distribution along the centerline upstream of the step was rather insensitive to variations in end-plate design.² Oil-flow visualization studies showed reasonable two-dimensionality within the separated region when the end plates extended well upstream of the separation line and enclosed the shear layer. Significant three-dimensional effects were observed in the oilflow patterns when smaller end plates were used, or when the step spanned the test section and interacted with the side-wall boundary layers.

Supersonic Test

One set of experiments was conducted in the Trisonic Four-Foot Wind Tunnel of the McDonnell Douglas Aerophysics Laboratory. In this case, air was injected through a slot installed in the north side-wall of the test section. Figure 2 is a sketch of the test region, including the array of static pressure orifices. The end plates were 10 in. high and had a leading-edge sweep angle of 60°. Nozzle inserts with slot widths of 0.053 and 0.50 in. and spoilers with heights of 2.0 and 4.5 in. were used.

Air was supplied through a large plenum chamber attached to the exterior of the tunnel wall. Interchangeable orifice meters were located at the junction of the air line and the plenum chamber. Pressure and temperature probes were located both in the jet air-supply line and in the plenum chamber. Discharge coefficients for a contoured-inlet meter were determined by the method of Stratford. Values of jet mass flow rate, determined by square-edged and by contoured-inlet orifice meters, agreed to within 2%.

Boundary-layer parameters were obtained from surveys on the south side-wall of the tunnel test section. Values of Re_L , the Reynolds number based upon the effective flat-plate distance from the origin of the boundary layer to the nozzle location, were computed from corresponding values of Reynolds number based on momentum thickness, and M_1 , the test-section Mach number.

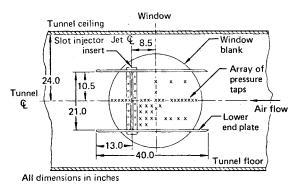


Fig. 2 Diagram of supersonic test region.

Static pressure data obtained from this experiment are presented in Figs. 3-5, in the form of P/P_1 vs x, measured from the center of the slot upstream along the tunnel centerline (see Fig. 2). Pressures measured at off-centerline locations were in good agreement with values obtained at the centerline. Data obtained with injection through the smaller

$$\begin{array}{c|c} & P_{0j}/P_1 \\ \hline o & \hline - \\ \Delta & 70.5 \\ \hline 0 & 125.1 \\ \Diamond & Spoiler, \ h = 4.5 \ in. \\ \hline M_1 = 2.50 \\ \hline \end{array}$$

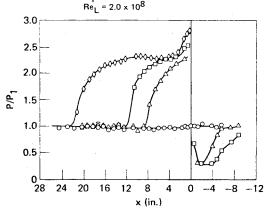


Fig. 3 Static pressure distributions, $M_1 = 2.50$, d = 0.053.

	P_{oj}/P_1	ReL
۵	254	3.0×10^{8}
Д	210	3.0 x 10 ⁸
Δ	201	5.6 × 10 ⁸
D	135.9	3.0×10^{8}
⊽	89.2	3.0×10^{8}
۵	52.5	3.0×10^{8}
♦	15.4	3.0×10^{8}
0	_	3.0 x 10 ⁸

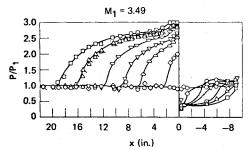


Fig. 4 Static pressure distributions, $M_1 = 3.49$, d = 0.053.

Fig. 5 Static pressure distributions, $M_1 = 3.49$, d = 0.50.

nozzle and with a spoiler at $M_1 = 2.50$ are presented in Fig. 3, and data obtained at Mach 3.49 with injection through the smaller nozzle are presented in Fig. 4. A set of data that was obtained at a higher Reynolds than the others is included in Fig. 4 for comparison.

The data of Fig. 5 were obtained with the wide slot and a correspondingly lower range of pressure ratios so that the range of jet mass flow rate covered by the data of Figs. 4 and 5 is about the same. At the lowest pressure ratio, the static pressure measured immediately upstream of the slot was too high to allow sonic flow to be maintained at the slot.

Hypersonic Tests

Two series of hypersonic experiments were conducted, one in the Hypervelocity Impulse Tunnel of the McDonnell Douglas Aerophysics Laboratory, and the other in the 96-in. leg of the Hypersonic Shock Tunnel of the Calspan Corporation.

Each model consisted of a splitter plate that extended from the tunnel nozzle throat to the test section and was then joined

<u>м</u> 1	ReL	Poj/P1
0 8.43	8.57 x 10 ⁷	474
0. 7.95	1.66 × 10 ⁷	421
o 8.45	7.30 × 10 ⁷	155

d = 0.10 in. Nozzle span = 14.0 in. End plate height = 12.0 in.

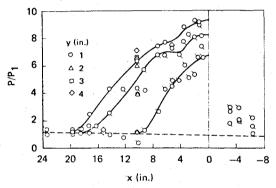


Fig. 6 Static pressure distributions, McDonnell Douglas hypersonic test, $M_1 \approx 8$. Test conditions are indicated by flags on symbols; spanwise locations are indicated by symbol shape.

M ₁	ReL	$\frac{P_{0j}/P_1}{}$
0 12.35	2.75 x 10 ⁷	495
Δ 12.38	2.92 x 10 ⁷	232
n 12.40	3.02 x 10 ⁷	580

Open symbols v = 0Closed symbols v = 6.0 in. d = 0.10 in.

Nozzle span = 24.0 in.

End plate height = 18.0 in.

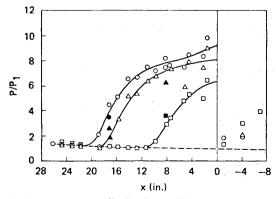


Fig. 7 Static pressure distributions, Calspan hypersonic test, $M_1 \simeq 12$.

to a shorter, instrumented plate. The arrangement in the vicinity of the jet nozzle was similar to that of Fig. 2. The nitrogen supply was controlled by a fast-acting valve in the McDonnell Douglas test, and by a charge tube and diaphragm assembly in the Calspan test. Primary data included test section flow conditions, jet reservoir conditions, and static pressure distributions on the plate in the vicinity of the jet. Jet mass flow rate was computed from the jet stagnation pressure and temperature, with an assumed nozzle discharge coefficient of unity. Some heat transfer measurements were made on the surface of the instrumented plate, without jet flow, during the McDonnell Douglas test. A comparison between measured and calculated values of heat transfer rate indicated that the boundary layer on the instrumented plate was turbulent at all test conditions. Since the Calspan tests were conducted at much higher Reynolds numbers, the boundary layers were presumed to be turbulent during these tests as well.

Examples of static pressure data are presented in Figs. 6 and 7. Pressures have been normalized by P_1 , the static pressure at the slot station corresponding to no jet flow. The dashed lines in these figures correspond to the static pressure distribution in the absence of jet flow. The Reynolds number was computed by multiplying the unit Reynolds number at the slot station by the distance from the tunnel nozzle throat to the slot.

The data of Fig. 6 correspond to a nominal test section Mach number of 8. These data possess the same general characteristics as the data obtained at lower Mach numbers, although the scatter is somewhat larger. This scatter is attributed to a combination of short test-time and unsteadiness of the interaction region.

Although little static pressure data were obtained downstream of the jets and no measurements were made in the immediate vicinity of the downstream lip of the nozzle, all of the downstream data show pressure levels somewhat higher than those of the undisturbed flow, but considerably lower than the pressure levels in the upstream region.

The data of Fig. 6 include a Reynolds number variation slightly greater than 5:1. This Reynolds number variation was accompanied by a significant variation in test-section Mach

Data corresponding to a nominal test-section Mach number of 12 are presented in Fig. 7. The shapes of the pressure distributions upstream of the jets are similar to the data ob-

	Source	Ref. No.
0	Supersonic test	
0	Hypersonic tests	_ '
\Q	Sterrett and Barber	4
0	Heyser and Maurer	5
4	Gray and Rhudy	6
D	Todisco and Reeves	7
£.	Elfstrom	8
0	Holden	9
0	Various sources identified	d in Ref. 10
Open symbols: separation produced by ramps, steps, or shock impingement		

Solid symbols: Jet-induced separation

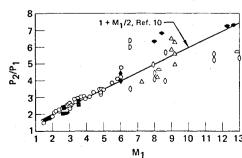


Fig. 8 Correlation of plateau pressure data for two-dimensional turbulent boundary-layer separation.

tained at supersonic Mach numbers and to the preceding Mach 8 data. These data also show pressures downstream of the jet that are greater than the values for the undisturbed flow. Data were obtained much closer to the downstream lip of the nozzle in the Calspan test than in the McDonnell Douglas test.

Discussion of Results

A summary of plateau pressure data for turbulent interaction with both jets and steps is presented in Fig. 8. ⁴⁻¹⁰ The data are restricted to those pressure distributions in which a well-defined plateau is formed. The empirical equation proposed by Zukoski ¹⁰ is also shown.

The results obtained with jets are self-consistent and agree well with the data in which separation was induced by steps, ramps, and impinging shocks. The supersonic data, $M_1 \approx 6$, show considerably less scatter and systematic variation in these coordinates than the hypersonic data, $M_1 \lesssim 6$. The variation in P_2/P_1 for supersonic turbulent boundary-layer separation with Reynolds number is small, except possibly near the transition Reynolds number. 10,11 A significant exception to this conclusion is a set of data presented by Werle et al. 12 at $M_1 = 4$, which show decreasing P_2/P_1 with increasing Reynolds number $(2 \times 10^5 \approx \text{Re}_{Ls} \approx 7 \times 10^6$, where L_c is the running length from the effective origin of the turbulent boundary layer to the spearation line). Supersonic data shown in Fig. 8 correspond to $10^4 \approx \text{Re}_{\delta 1} \approx 5 \times 10^6$. Considerable scatter and systematic variation are present in the hypersonic data. Some of the scatter is probably a result of the inherently poorer accuracy and repeatability of shock tunnels and gun tunnels, relative to continuous and blowdown tunnels. In a review of the hypersonic data by Reeves,13 it was concluded that P_2/P_1 becomes increasingly sensitive to the length of the separated shear layer with increasing M_1 .

Results of studies reported by Zukoski, ¹⁰ Werle et al.; ¹² and Driftmyer, ¹⁴ agree concerning the length scales for pressure distributions resulting from jets or solid obstructions. They indicate that δ_1 is the appropriate length scale for the static pressure distribution in the region beginning with the initial pressure rise and extending somewhat downstream of the location of maximum pressure gradient. Further

	M ₁	δ ₁ (in.)	Re _{δ 1}
0	4.46	3.17	3.52 x 10 ⁶
A	2.50	2.14	2.16×10^{6}
Δ	2.50	2.11	2.14×10^{6}
٥	2.50	2.04	2.06×10^{6}
۵	3.49	2.58	5.04 x 10 ⁶
D	3.49	2.79	3.14×10^{6}
0	3.49	2.78	3.10×10^{6}
0	12.35	4.0	0.72×10^{6}
Δ	12.40	4.0	0.80 × 10 ⁶

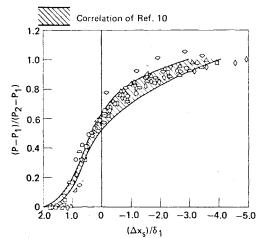


Fig. 9 Correlation of static pressure distribution for turbulent boundary-layer separation.

downstream, the dominant length scale changes to the separation distance, step height, or effective jet penetration height. Data obtained from the present study are in agreement with these results, as shown in Fig. 9. The solid lines represent the range of variation of the data presented in Ref. 10, which correspond to $h/\delta_1 > 1$ and $2 \le M_1 < 6$. In this case, the origin has been located at the point $P-P_1=0.6$ (P_2-P_1). It is clear that any choice of origin that would cause superposition of the regions of maximum pressure gradient would show the same result.

The hypersonic data are in reasonable agreement with the correlation when $(P-P_1)/(P_2-P_1) \le 0.6$, but when $(P-P_1) > 0.6$, the data lie above the primary correlation curve. This behavior has been noted as a characteristic of data obtained with forward-facing steps when $h/\delta_1 < 1$, and is, therefore, consistent with the fact that the hypersonic data correspond to effective values of h less than δ_1 .

The interaction force developed upstream of a two-dimensional, normal, sonic jet in turbulent flow has been the subject of many investigations $^{4,11,12,15\cdot18}$. Figure 10 summarizes data in the form of the amplification factor, K, vs M_1 for air or nitrogen jets. Amplification factor is defined as the upstream interaction force plus the jet thrust normalized by the vacuum thrust of a sonic jet having the same stagnation conditions and mass flow rate as the actual jet. Interaction forces were determined by integrating static pressure distributions upstream of the slot along the x-axis, except for the data of Hawk and Amick, 17 which were obtained by direct measurement of forces.

Many of the sets of data shown in Fig. 10 were obtained from flat-plate experiments at constant external flow conditions, fixed slot width, and varying P_{0j}/P_1 . As a result, variations in pressure ratio also correspond to variations in jet mass flow rate, size of the effective obstruction produced by the jet, Reynolds number at separation, distance between transition and separation, aspect ratio of the separated region, etc. Data taken in this manner always show a decrease in amplification factor with increasing P_{0j}/P_1 .

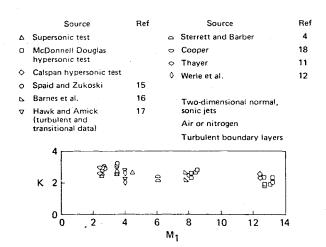


Fig. 10 Effect of external flow Mach number on jet interaction performance.

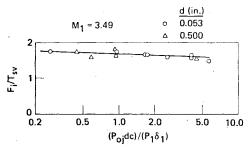


Fig. 11 Effect of pressure ratio and slot width on interaction force.

Although substantial variation in the data is present at a fixed Mach number, the data indicate that the influence of M_1 on K is quite small. This behavior is in agreement with predictions of several semiempirical theories. ^{12,15,16} Data from the present study and from Ref. 15 are the only sets of data obtained with large end plates that include measurements made at several values of M_1 in the same facility and with the same model and instrumentation. Examination of these data separately also supports the conclusion that the influence of M_1 on K is small.

The data of Fig. 10 cover a wide range of effective flat-plate Reynolds number, 2.0×10^6 to 5.5×10^8 . The extremes in this Reynolds number range are represented by data from different facilities. Data from a given test series in which P_{0i} is varied correspond to variation in Re_{Ls}; however, corresponding variations in K cannot definitely be attributed to a Reynolds number effect, for reasons which have already been discussed. Although some differences exist between sets of data obtained from different facilities at the same Mach number and at different Reynolds numbers, no consistent performance trend with Reynolds number has been found. Data presented by Thayer¹¹ in the form of K vs Re_{Ls} (0.4 × 10⁶ < $Re_{Ls} < 2.5 \times 10^6$) show the effect of Reynolds number to be very small with nitrogen or helium as the injectant, but data obtained with hydrogen as the injectant show an 11% increase with increasing Re_{Ls} within the same range. A set of data obtained at $M_1 = 4$ by Werle et al., 12 in which the freestream unit Reynolds number was changed by a factor of 3:1 and data from the present study corresponding to a 5:1 variation at $M_1 \approx 8$, did not show a significant effect of Reynolds num-

The decrease in K with increasing P_{0i}/P_1 for a fixed slot width, d, and a fixed external flow has often been interpreted as an effect of pressure ratio, but the independent effect of pressure ratio can only be determined with confidence from experiments in which effects of other variables can be evaluated, i.e., experiments in which the slot width is changed. A plot of F_i/T_{sv} vs $(P_{0j}dc)/(P_1\delta_1)$ is shown in Fig. 11, using data from the present study. The quantity c is the discharge coefficient for the slot. The value of δ_1 is essentially constant in this case. The boundary-layer thickness was chosen for normalization because it is a characteristic length corresponding to the external flow. These data have been used for comparison because they include a large variation in slot width and accurate measurements of jet mass flow rate. Jet mass flow rate measurements independent of those computed from the slot area and jet stagnation conditions are highly desirable. First, the slot area is seldom known to the required accuracy even in the absence of jet flow effects. Second, slot geometries and Reynolds numbers are usually such that the discharge coefficients are significantly different from unity and dependent upon Reynolds number. Third, slots have been known to deform enough at high values of jet stagnation pressure to cause large changes in nozzle area. 19 The data of Fig. 11 are almost completely independent of pressure ratio or slot width for $3.26 \le P_{oj}/P_1 \le 254$. These data span the range of subsonic to highly underexpanded jet flow at the nozzle exit. Data reported by Werle et al 12 show similar results.

Conclusions

The following conclusions are limited to two-dimensional flows with fully developed turbulent bounday layers upstream of separation. Variations in Mach number, Reynolds number, or presssure ratio have little influence on the upstream amplification factor. Data in which a variation of Mach number was obtained for a fixed set of experimental equipment show a smaller variation of amplification factor with Mach number than variations between results of different experiments. The effect of pressure ratio was investigated at constant external flow conditions and jet mass flow rate.

Pleateau pressure levels for separated flows at Mach 2.50 and 3.49 and Reynolds numbers based on running length greater than 10⁸ agree well with data from similar experiments at much lower Reynolds numbers. Results of the present experiments are in good agreement with a previous correlation of static pressure distributions in the vicinity of separation, which indicates that the boundary-layer thickness for the undisturbed flow is the most important length scale in this region.

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